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CALCULATION OF THE NONLINEAR AERODYNAMIC CHARACTERISTICS OF A WING OF FINITE SPAN

V. A. Algazin

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Problems of setting up the methods of calculation and the calculation of a flow around thin wings of finite span moving with large angles of attack in an ideal incompressible liquid were considered in [1-5]. Common to all methods is successive linearization of the problems with respect to time and modelling of the wing and the shroud behind it by vortical surfaces. In [1-3] these surfaces are replaced by a discrete system of vortical segments of constant intensity. In [4] the surface modeling the wing is replaced by a system of vortex rings which is analogous to the system being used in [1], while the account of the vortex shroud is based on the spatial discretization of the vortex vector which varies with the duration of time in accordance with the Helmholtz equation. At the basis of the algorithm [5] there lies a spline approximation of the intensity of the vortical surface by a function whose form takes into account the singularities of the flow close to the edges of the wing.

In the present work we have obtained a general system of nonlinear equations of the problem of flow around a wing of finite span moving in an ideal incompressible liquid from the state of rest. This system is solved by successive linearization [1-5] for a series of discrete time instants. The coordinates of points of the vortex shroud are determined, in contrast to [1-5], according to a difference expression of the second order. The solution of the linear problem (on each step in time) is constructed by means of the method of [5] which is modified so that the approximation of the intensity of the vortical layer by spline functions of special form is used only when establishing a connection between the various components of the discrete singularities.

The numerical calculations have been carried out within the framework of a model which takes into account the vortex shroud emerging only from the rear edge of the wing. The convergence of the method within the framework of this model was established numerically. The problem concerned with the influence of the order of approximation of the intensity of the vortical layer and the magnitude of the step in time on the stability of computation is considered; the structure of the vortex shroud behind the wind and its influence on the aerodynamic characteristics of rectangular wings with different lengthening, and also dependence of the force of drag and the efficiency of a waving wing on the Strouhal number are investigated.

1. We consider the motion of a thin wing of finite span in an ideal incompressible liquid. We introduce the right-handed rectangular system of dimensionless (referred to the length of root chord b of the wing) coordinates $O_1x_1y_1z_1$, at an infinitely remote point of which the liquid is at rest. Let at the instant of time $\tau = 0$ the wing begin motion from the state of rest at a certain given velocity $V(x_1, y_1, z_1, t)$, where $t = V_0\tau/b$, while V_0 is

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a certain characteristic velocity (e.g., $V_0 = |\mathbf{V}(\tau_*)|$, $\tau_* > 0$). We denote the surface of the wing and the vortex shroud behind it by $S_0(t)$ and $S_1(t)$, respectively the front edge of the wing surrounded by a flow without separation by L_s, and the part of the edge from which the vortex shroud emerges by L_w. We assume that the motion of the liquid outside the surface $S = S_0 \cup S_1$ is potential.

The surface S(t) will be modeled by a vortical surface with the intensity

$$\boldsymbol{\gamma} = \mathbf{v} \times (\mathbf{v}_+ - \mathbf{v}_-),$$

while the jump in the pressure p at a point $M \in S(t)$ will be determined by means of the Cauchy–Lagrange integral

$$\frac{p_- - p_+}{\rho V_0^2} = \frac{\partial}{\partial t} \int\limits_{L(M)} (\boldsymbol{\gamma} \times \mathbf{v})\, d\mathbf{r} + (\boldsymbol{\gamma} \times \mathbf{v})(\mathbf{v}_0 - \mathbf{v}_e), \tag{1.1}$$

where by the indices plus and minus we have denoted the limit values of the functions when approaching the surface S from above and from below respectively; $\mathbf{v}$ is the base vector of the normal to the upper side of this surface; ρ is the density of the liquid; $\mathbf{v}_0 = (\mathbf{v}_+ + \mathbf{v}_-)/2$; $\mathbf{v}_e$ is the transport velocity of the point M under consideration.

We assume that at each time instant t the surface S(t) is smooth in the sense of Lyapunov, while the vector-function $\gamma(M, t)$ on it belongs to the class H* [6] in the neighborhood of the edge L_s. This allows us to determine the velocity at any point of the liquid and at a point $M \in S(t)$ by means of the well-known expression of Biot–Savart. The field of velocities thus obtained is potential outside S(t), and the perturbed velocities are damped out at infinity everywhere outside $S_1(t)$. Satisfying the remaining conditions of the problem of flow around a thin wing of finite span (see, e.g., [1, 5]) for the intensities γ_0, γ_1 of the vortex layers on the surfaces $S_0(t)$, $S_1(t)$ and coordinates of the vortex shroud, we obtain the system of equations for $M \in S_0(t)$

$$\iint\limits_{S_0} \frac{(\boldsymbol{\gamma}_0 \times \mathbf{R})\,\mathbf{v}}{R^3}\, dS = 4\pi \mathbf{V}\cdot\mathbf{v} - \iint\limits_{S_1} \frac{(\boldsymbol{\gamma}_1 \times \mathbf{R})\,\mathbf{v}}{R^3}\, dS; \tag{1.2}$$

$$\operatorname{div} \boldsymbol{\gamma}_0 = 0; \tag{1.3}$$

for $M \in S_1(t)$

$$\partial \mathbf{r}/\partial t = \mathbf{v}_0(\mathbf{r}, t), \quad \mathbf{r}(\boldsymbol{\gamma}, t_\gamma) = \mathbf{r}_0(\boldsymbol{\gamma}); \tag{1.4}$$

$$\begin{cases} (\boldsymbol{\gamma}_1 \cdot \mathbf{i}_1)\sqrt{g} = \Phi_1 \sqrt{g_{vv}} \cos\psi + \Phi_2 \sqrt{g_{uu}}, \\ (\boldsymbol{\gamma}_1 \cdot \mathbf{i}_2)\sqrt{g} = \Phi_1 \sqrt{g_{vv}} + \Phi_2 \sqrt{g_{uu}} \cos\psi; \end{cases} \tag{1.5}$$

$$\operatorname{div} \boldsymbol{\gamma}_1 = 0; \tag{1.6}$$

$$\frac{d}{dt} \int\limits_{L(M)} (\boldsymbol{\gamma}_0 \times \mathbf{v})\, d\mathbf{r} = (\mathbf{w} \times \mathbf{v})\,\boldsymbol{\gamma}_1, \quad \mathbf{w} = \mathbf{v}_0 - \mathbf{V},\ M \in L_w, \tag{1.7}$$

where $\mathbf{r} = \mathbf{r}(\gamma, t)$ is the radius vector of points of the free vortical surface $S_1(t)$, being considered as a function of vorticity $\boldsymbol{\gamma}$ and the time t; t_γ is the instant of departure of the vortex $\boldsymbol{\gamma}$; from the edge L_w; $\mathbf{r}_0(\boldsymbol{\gamma})$ is the radius vector of this vortex at $t = t_\gamma$; L(M) is an arbitrary curve joining the point $M \in L_w$ with a point on the boundary $S_0(t)$ to the edge L_s; $\mathbf{i}_1$, $\mathbf{i}_2$ are the base vectors of the coordinate base of the surface $S_1(t)$; ψ is the angle between the coordinate lines u = const, v = const; g_{uu}, g_{uv}, g_{vv} are the Gaussian coefficients of the surface $S_1(t)$, $g = g_{uu}g_{vv} - g_{uv}^2$; the quantities $\Phi_1(u, v)$, $\Phi_2(u, v)$ are determined at the instant of formation of the vortex $\gamma_1(u, v, t)$ and in the following retain constant values for fixed u, v on $S_1(t)$ although the surface itself deforms in accordance with variation of the velocity field.

Since the region of flow of the liquid and the velocity vector $\mathbf{V}$ of the motion of points of the wing depends on time, the system (1.2)-(1.7) must be solved with initial data which in the case of motion from the state of rest has the form

$$S(0) = S_0(0), \ \gamma(M, 0) = 0. \tag{1.8}$$

The solution of the system (1.2), (1.3) for γ_0 will be sought in the class of functions satisfying on the edge L_s the condition that the component of intensity of the vortical layer normal to this edge is zero

$$\gamma_0 \cdot \tau = 0, \ M \in L_s. \tag{1.9}$$

Here τ is the base vector of the tangent to the surface $S_0(t)$ on L_s surrounded by flow without separation.

2. We shall solve the system (1.2)-(1.7) for a series of discrete time instants t_n, commencing from $t_0 = 0$, for which the conditions (1.8) are fulfilled.

We assume that at the time instant $t = t_n$ the solution of the system is known, and obtain its solution for $t = t_{n+1} = t_n + \Delta t_{n+1}$. We introduce a coordinate system $y = 0$ to coincide with the plane of right half-wing $S_0^-(t_{n+1})$. The axis Ox is directed from the front edge backward along the root chord, and the axis Oz to the left along the span. We assume that the law of motion of the wingis symmetric relative to the Oxy plane. In this case for the calculation of the loads on the wing it is sufficient to determine γ_0 only on one half of the wing, for example, S_0^-.

The surface of the half wing S_0^- is divided into n_x equal strips along the chord, and into n_z strips along the span. At the intersections of these strips we obtain $N = n_x n_z$ elements S_{0q}. Application of a step procedure in time for the solution of the system (1.2)-(1.7) leads to the fact that for $t = t_{n+1}$ the surface of the right half of the vortex shroud S_1^- will be divided into $N_1 = (n + 1)N_s$ elements S_{1q}, where $N_s = n_x + n_z$. The elements $S_{1q}(q = nN_s + 1, \ldots, N_1)$ being formed over the time Δt_{n+1} as a result of exit of vortices from the wing, with an accuracy up to quantities of the order $(\Delta t_{n+1})^2$ are located in the plane of the wing. The totality of these elements is denoted by $\Delta S_1^-(t_{n+1})$.

With each element $S_{lq}(l = 0, 1)$ of the vortical surface we match the vector $\mathbf{\Gamma}_{lq}^{(n+1)}$ of the summary intensity of the vortex layer γ_l, assuming [5]

$$\mathbf{\Gamma}_{lq}^{(n+1)} = \iint_{S_{lq}} \gamma_l(t_{n+1})\, dS. \tag{2.1}$$

On the surface $S_0^- \cup \Delta S_1^-$ the vectors $\mathbf{\Gamma}_{lq}$ have two components Γ_{lxq}, Γ_{lzq}, while on $S_1^- \backslash \Delta S_1^-$, generally speaking, they have three components which are not zero at the same time. The components Γ_{orq} $(r = x, z)$ of the vectors $\mathbf{\Gamma}_{oq}$ are placed at the center of gravity (x_{rq}, z_{rq}) of elements S_{oq} whose density is γ_{or} [7], while $\mathbf{\Gamma}_{1q}$ are connected with points $(x_q, y_q, z_q) \in S_{1q}$ whose radius vectors in the Oxyz system are denoted by $\mathbf{r}_q^{(n+1)}$.

Taking into account (1.4)-(1.6), we can show that the position $\mathbf{r}_q^{(n+1)}$ and the magnitude of the vectors $\mathbf{\Gamma}_{1q}^{(n+1)}$ $(q = 1, \ldots, nN_s)$, which have been formed up to the time instant t_{n+1} are completely determined by the solution of the problem for $t < t_{n+1}$. The remaining $N + N_s$ of the vectors $\mathbf{\Gamma}_{0q}^{(n+1)}$ $(q = 1, \ldots, N)$ and $\mathbf{\Gamma}_{1q}^{(n+1)}$ $(q = nN_s + 1, \ldots, N_1)$, which schematize the surface $S_0^-(t_{n+1}) \cup \Delta S_1^-(t_{n+1})$, depend on the solution at the time instant under consideration. To determine them, from the system of equations (1.2), (1.3), (1.6), and (1.7) we go to the system of algebraic equations relative to the $2N + 2N_s$ quantitites Γ_{oxq}, Γ_{ozq}, Γ_{1qx}, Γ_{1qz}.*

From the integral equation (1.2), satisfying it at N check points and using the schematization ofthe vortical surface $S(t_{n+1})$ by the discrete system of vectors (2.1), we obtain N linear algebraic equations

$$\sum_{q=1}^{N} (\Lambda_{0xiq}\Gamma_{0xq} + \Lambda_{0ziq}\Gamma_{0zq}) + \sum_{s=1}^{N_s} (\Lambda_{1xis}\Gamma_{1xq} + \Lambda_{1zis}\Gamma_{1zq}) =
= 4\pi V_y(X_i, Z_i, t_{n+1}) - \sum_{m=1}^{nN_s} (L_{1im}C_{1m} + L_{2im}C_{2m}), \tag{2.2}$$

*In what is to follow, when it is clear that the quantities being considered refer to the time instant t_{n+1}, the index at the top will be omitted.

where

$$\Lambda_{lriq} = F_{lriq} \pm \Delta F_{lriq}; \quad L_{pim} = K_{pim} + \Delta K_{pim} \quad (l = 0, 1, \ p = 1, 2);$$
$$C_{1m} = \Phi_{2m}(v_{2m} - v_{1m})\, l u_m^{(n+1)}; \quad C_{2m} = \Phi_{1m}(u_{2m} - u_{1m})\, l v_m^{(n+1)};$$

$s = q - nN_s$, while the sign plus (minus) corresponds to $r = z$ $(r = x)$. Here

$$\begin{cases} F_{0xiq} = \dfrac{\zeta_{xiq}}{(\xi_{xiq}^2 + \zeta_{xiq}^2)^{3/2}}, \quad F_{0ziq} = \dfrac{\xi_{ziq}}{(\xi_{ziq}^2 + \zeta_{ziq}^2)^{3/2}}, \\ K_{pim} = \dfrac{\xi_{im} i_{pmz} - \delta \zeta_{im} i_{pmx}}{(\xi_{im}^2 + y_m^2 + \zeta_{im}^2)^{3/2}}, \quad \delta = 1; \end{cases} \tag{2.3}$$

$$\xi_{riq} = X_i - x_{rq}, \ \zeta_{riq} = Z_i - \delta z_{rq}, \ \delta = 1, \tag{2.4}$$

ξ_{im}, ζ_{im} coincide with (2.4) in the case of $x_{rq} = x_m$, $z_{rq} = z_m$, where (x_m, y_m, z_m) is the coordinate of the point $\mathbf{r}_m^{(n+1)}$ at which the singularity $\Gamma_{1m}^{(n+1)}$ is located; i_{lrq} are the projections of the base vectors $\mathbf{i}_{lq}$ of the coordinate base of the element $S_{1q}^{(n+1)}$ at the point $\mathbf{r}_q^{(n+1)}$ onto the Or(r = x, z) axis; u_{lq}, v_{lq} $(l = 1, 2)$ are the boundaries of the element S_{1q} (u, v, t); $l u_q^{(n+1)}$ $(l v_q^{(n+1)})$ is the length of the coordinate line $v = v_q = (v_{1q} + v_{2q})/2$ $(u = u_q = (u_{1q} + u_{2q})/2)$ for $t = t_{n+1}$; $\Phi_{lq} = \Phi_l(u_q, v_q)$; the quantities ΔF_{orq}, ΔL_q take into account the influence of the left half of the vortical surface and are obtained from (2.3), (2.4) for $\delta = -1$; the expressions for F_{1rq}, ΔF_{1rq} coincide with the corresponding expressions for F_{orq}, ΔF_{orq} in the case $x_{rq} = x_q$, $z_{rq} = z_q$. The check points X_i, Z_i in (2.2) are chosen from the condition of minimum quadratic deviation of the velocity induced by the system of singularities (2.1) [7]. We also note that by the choice of the position of the discrete singularities (2.1) on the wing S_0 and the part ΔS_1 of the shround S_1 we ensure fulfilment of the Zhukovskii hypothesis about finiteness of the velocity at points of the edge L_w and the condition $\gamma=0$ at the points of break of the wing contour in plan [5,7].

From Eq. (1.7), taking into account (1.5), we go to the system of N_s equations for the intensities of the vortices rolling off

$$\sum_{k=1}^{n_x} \left(\Gamma_{0zq}^{(n+1)} - \Gamma_{0zq}^{(n)}\right) = -\Gamma_{1zi}^{(n+1)} + \frac{w_{zi}^{(n)}}{w_{xi}^{(n)}} \Gamma_{1xi}^{(n+1)}, \quad i = N_2 + j, \ j = 1, \ldots, n_z; \tag{2.5}$$

$$\sum_{j=1}^{n_z} \left(\Gamma_{0xq}^{(n+1)} - \Gamma_{0xq}^{(n)}\right) = -\frac{w_{xi}^{(n)}}{w_{zi}^{(n)}} \Gamma_{1zi}^{(n+1)} + \Gamma_{1xi}^{(n+1)}, \quad i = N_1 - k + 1, \ k=1, \ldots, n_x, \tag{2.6}$$

where $q = (k - 1)n_z + j$.

The system of $N + N_s$ equations (2.2), (2.5), (2.6) is not closed, since the number of equations is less than the number of unknowns. To obtain yet $N + N_s$ equations we use the conditions (1.3), (1.6) and the approximation of the intensity of the layer γ_0 on S_0 and γ_1 on ΔS_1 by spline functions of a special form which take into account the singularities of the flow on the wing edge L_s, which is described in [5]. Let the component γ_{0z} (γ_{1z}) of the intensity of the vortex layer on the element S_{0q} $(S_{1q} \in \Delta S_1)$ (in terms of the paper [5]) by the function $\gamma_{lz}^{(q)}$ $(l = 0,1)$. The component $\gamma_{lx}^{(q)}$ is determined from the solution of Eqs. (1.3), (1.6) with the boundary condition (1.9), which in the given case assumes the form $\gamma_{0x}(0, z, t) = 0$. According to the definition (2.1) we have

$$\Gamma_{lrq} = \iint_{S_{lq}} \gamma_{lr}^{(q)} dx dz + \Delta_{lrq}, \quad l = 0, 1, \ r = x, z, \tag{2.7}$$

where $\Delta_{lrq} = \iint_{S_{lq}} (\gamma_{lr} - \gamma_{lr}^{(q)})\, dx dz$ is the error of approximation on S_{lq}. The integral in (2.7) depends on the values of the functions γ_{0z}, γ_{1z} at the corner points of the wing and the element of the trace adjoining the edge L_w [5]. Eliminating them and neglecting the quantities Δ_{lrq},* from (2.7) we obtain $N + N_s$ additional equations

*It can be shown that in the case of uniform partition of a rectangular wing the elongations λ into n_z strips over the half-span are $|\Delta_{lrq}| \leqslant C_r\lambda/(2n_z\sqrt{n_x})$.

$$\Gamma_{0x} = D_0\Gamma_{0z} + D_1\Gamma_{1z} + d_0, \quad \Gamma_{1x} = G_0\Gamma_{0z} + G_1\Gamma_{1z} + d_1. \tag{2.8}$$

Here Γ_{0r}, Γ_{1r} are algebraic vectors of the length N and N_1 formed from quantities Γ_{0rq}, Γ_{1rq} respectively; the matrices D_0 and G_0 are square, of the orders N and N_s, while D_1 and G_1 are rectangular, of the dimensions $N \times N_s$ and $N_s \times N$; the vectors d_0, d_1 have the lengths N and N_s, and depend on the solution at the time instant t_n, where in view of (1.9) $d_0(t_0) = d_1(t_0) = 0$. The relations (2.8) are a discrete analog of the conditions (1.3), (1.6).

The system of linear algebraic equations (2.2), (2.5), (2.8) is now closed. Its solution determines the quantities $\Gamma_{lrq}^{(n+1)}$, commencing from n = 0. The values thus found allow us to determine the continuous vortex layer on S_0^- [5] which is required in the calculation of the distributed loads on the wing.

3. The normal force P_q (referred to $\rho V_0^2 b^2/2$) acting on the element S_{0q}, in accordance with (1.1) ($\mathbf{v}_e = \mathbf{V}$) we represent in the form

$$P_q \equiv \iint\limits_{S_{0q}} dP = P_{qx} + P_{qz} + P_{qt} + P_{qit}, \tag{3.1}$$

where P_{qr} (r = x, z) determine the part of the force depending on γ_{0r}, while the quantities P_{qt}, P_{qit} are connected with the circulation over the j-th strip of the half-wing

$$P_{qt} = -2h\frac{d}{dt}\sum_{m=1}^{k-1}\Gamma_{0zi}^{(n+1)}, \quad h = \frac{1}{n_x}, \qquad i = (m-1)\,n_z + j,$$

$$P_{qit} = -2\frac{d}{dt}\iint\limits_{S_{0q}}\left(\int\limits_{(k-1)h}^{x}\gamma_{0z}d\xi\right)dxdz.$$

Discarding quantities of the order $h\Gamma_{0rq}$, $\sigma_j\Gamma_{0rq}$ and above, we obtain

$$P_{qx} = W_{qz}\Gamma_{0xq}^{(n+1)}, \quad P_{qz} = -W_{qx}\Gamma_{0zq}^{(n+1)},$$
$$P_{qit} = -2h(1-\mu_{xzk})\frac{d}{dt}\Gamma_{0zq}^{(n+1)},$$

where $\sigma_j = z_{j-1} - z_j$ is the width of the j-th strip along the span, the coefficient μ_{xrk} determines the position ($x_{rq} = (k-1+\mu_{xrk})h$) of the discrete singularity Γ_{0rq} on the element S_{0q} in terms of fractions of its linear dimensions, while

$$W_{qr} = 2w_r(x_{0q}, z_{0q}, t_{n+1}), \quad x_{0q} = (k-0.5)h, \; z_{0j} = 0.5(z_{j-1} + z_j).$$

The moment of the hydrodynamic forces acting on the element S_{0q}, about the front edge of the wing, isrepresented analogously to (3.1) in the form

$$M_{zq} = M_{zqx} + M_{zqz} + M_{zqt} + M_{zqit}.$$

Each component M_{zqr}(r = x, z, t, it) will be considered as the moment of the corresponding force P_{qr} which is applied to the element S_{0q} at the point

$$x = (k-1+\varkappa_{rq})h, \; z = z_{0j}, \; 0 < \varkappa_{rq} \leqslant 1.$$

Neglecting quantities of the order $h^2\Gamma_{0rq}$, $h\sigma_j\Gamma_{0rq}$ (r = x, z) and higher, we obtain

$$\varkappa_{xq} = \mu_{xxk}, \varkappa_{zq} = \mu_{xzk}, \varkappa_{tq} = 0.5, \varkappa_{itq} = (5 - 4\mu_{xzk})/(8(1-\mu_{xzk})).$$

The elemental inflow force (referred to $\rho V_0^2 b^2/2$) is obtained from the theorem of momentum change, applying it to the volume liquid with a sphere of radius $\varepsilon \ll 1$ with the center at the point (0, z) of the edge L_s of the wing. It can be shown that for $\varepsilon \to 0$

$$dQ = -\pi a^2(z)dz/2, \tag{3.2}$$

where $a(z)$ is the coefficient at the singularity $x^{-1/2}$ of the component γ_{0z} of intensity of the vortex layer. The approximation of the vortex layer proposed in [5] allows us to compute $\alpha(z)$ in terms of the quantity $\Gamma_{0zq}^{(n+1)}$. Integrating (3.2) along the entire edge L_s, we obtain the inflow force Q acting on the entire wing.

The dimensionless coefficients of the normal force P, the inflow force Q, and the moment M_z are determined as follows:

$$c_n = \frac{b^2 P}{S} = \frac{2}{\lambda} \sum_{q=1}^{N} P_q, \quad c_q = \frac{b^2 Q}{S}, \quad m_z = \frac{b^2 M_z}{S} = -\frac{2}{\lambda} \sum_{q=1}^{N} M_{zq}, \tag{3.3}$$

where S is the wing area.

4. In the case of practical implementation of the method presented in Secs. 2 and 3, the algorith of calculation of the aerodynamic characteristics is conditionally divided into a series of stages: 1) selection of the step in time Δt_{n+1}; 2) determination of the coordinates $\mathbf{r}_q^{(n+1)}$ of the surface S_1^- from the solution of the Cauchy problem (1.4) for the q-th singularity

$$\mathbf{r}_q^{(n+1)} = \mathbf{r}_q^{(n)} + \mathbf{w}_{1q}^{(n)} \Delta t_{n+1},$$

where

$$\mathbf{w}_{1q}^{(n)} = \begin{cases} \mathbf{w}_q^{(n)} \left(1 + \frac{\Delta t_{n+1}}{2\Delta t_n}\right) - \mathbf{w}_q^{(n-1)} \frac{\Delta t_{n+1}}{2\Delta t_n}, & q \leqslant nN_s, \\ \mathbf{w}_q^{(n)}, & q = nN_s + 1, \ldots, (n+1) N_s; \end{cases}$$

3) determination of the velocity induced on the wing by the system of discrete singularities which were formed up to the time instant $t_n + \Delta t_{n+1}$ from the expressions (2.3), (2.4); 4) computation of the connection matrices from (2.8); 5) determination of the quantities Γ_{lrq} (l = 0, 1; r = x, z) from the solution of the system (2.2), (2.5), (2.6), (2.8); 6) calculation of the field of velocities $\mathbf{w}_q^{(n+1)}$ at given points of the surface $S_0^- \cup S_1^-$ and determination of the coefficients c_n, m_z, c_q.

We shall consider certain stages of the calculation. In [8], when solving the problem of flow without separation around a wing of an infinite span, the step in time $\Delta t_{n+1} = t_{n+1} - t_n$ was chosen from the condition

$$\Delta t_{n+1} = 1/(n_x w_x(t_n)), \tag{4.1}$$

where w_x is the relative velocity at a point of the rear edge. The condition (4.1) ensures uniformity of the distribution of vortices in the neighborhood of the rear edge of the profile from which the vortex shroud emerges.

In the calculations to be presentedbelow, Δt_{n+1} is chosen from the condition (4.1), where in the role of w_x at any time instant t_{n+1} we have the velocity value $\langle w_x \rangle$ averaged over the length of the rear edge. The subsequent three stages (2-4) do not present major difficulties. The determination of the quantities Γ_{lrq} (stage 5) on $S_0^- \cup \Delta S_1^-$ in the framework of a model which takes into account the vortex shroud running off all edges of the wing, except the front edge L_s, is connected with elimination of a series of difficulties of a methodological character. In particular, when considering nonlinear nonstationary problems in the framework of this model, the magnitudes of the elements being formed over the time Δt_{n+1}, close to the side and rear edges of the wing, in a number of cases are different. This leads to nonuniformity in the location of the discrete singularities close to these edges and, as a consequence, to insufficient accuracy in the determination of their intensity. In the given paper the stage 5 and the subsequent examples of a numerical calculation are considered without taking into account the side vortex shroud. The solution of the system (2.2), (2.5), (2,6), (2.8) was determined on a BÉSM-6 computer by means of the generalized method of elimination of Gauss. From the known values of Γ_{lrq} we determined the velocity field and carried out the calculation of the aerodynamic characteristics. After this, transition was effected to the next step in time.

The convergence of the algorithm proposed above was verified numerically by comparing the results of the calculation with different number N of vortices on the half-wing. The

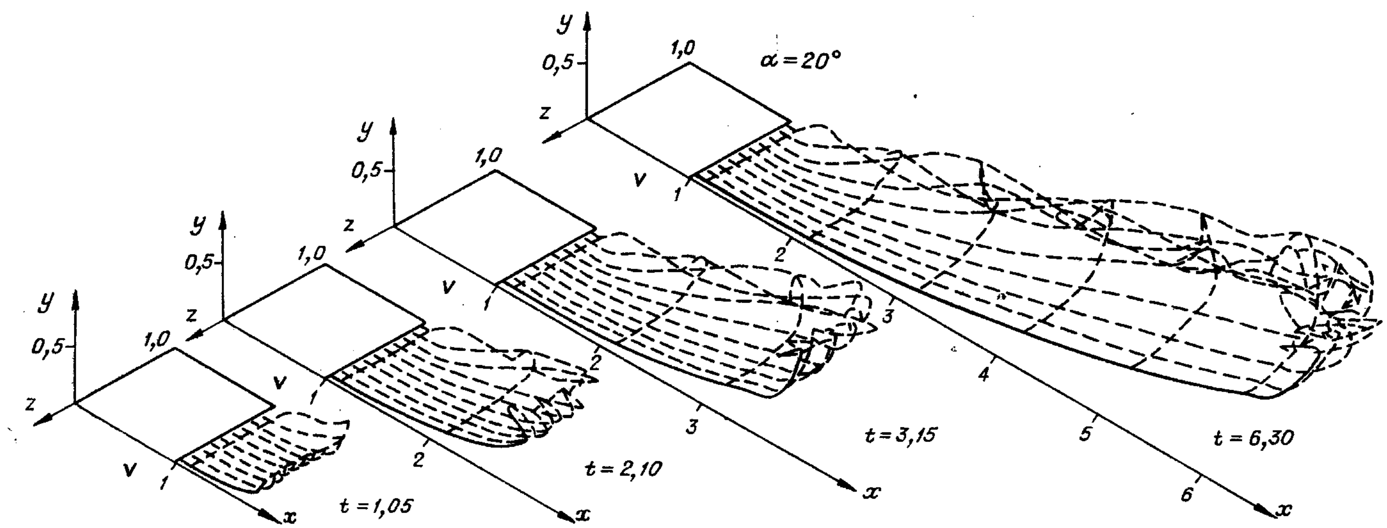

Fig. 1

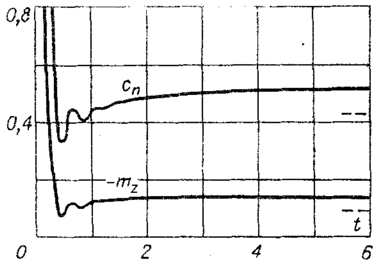

Fig. 2

results of the calculation with a step in time Δt_n and with a step in time equal to half of this (in view of (4.1) this corresponds to the fact that if the number of elements along the chord in one case is n_x, then in the other case it will be $2n_x$) practically coincided. This allows us to draw the conclusion about the convergence of the method in the given case.

We investigated the influence of the order of approximation of the vortex layer on the elements ΔS_{1q} of the vortex shroud, on the stability of the computation of the aerodynamic characteristics. In [5] it was proposed to approximate γ_{0z} on the wing close to the edge L_w by a polynomial of the second degree in x, and γ_{1z} on ΔS_1 by a linear function. The results of calculation of a number of wings with such an approximation showed that the computation becomes unstable. The instability of the calculation is connected, apparently, with the fact that variation of the matrices D_l, G_l (l = 0.1) in (2.8) alters the conditioning of the matrix of the entire system (2.2), (2.5), (2.6), (2.8). Therefore in the following, on ΔS_1 we took approximation of the same order as on the wing.

When computing with the step in time $\Delta t_{n+1} = \Delta t$ = const we investigated the influence of the quantity $\varepsilon_t = \Delta t_{n+1}/(n_x\langle w_x(t_n)\rangle)$ on the accuracy and stability of the calculation of the loads on the wing. It turned out that in the case $\varepsilon_t > 1$ (max $(\varepsilon_t) = 2$) the computation is stable. The coefficients (3.3) of forces and moments obtained as a result of such a calculation, for t greater than a certain t_*, practically coincide with the corresponding quantities in the case of $\varepsilon_t = 1$. As for the calculation in the case of $\varepsilon_t < 1$, then the computation becomes unstable after 5-10 steps in time, if $\varepsilon_t < 0.8$.

5. We now present certain results of the calculations. In Fig. 1 we have depicted the right half of the vortex shroud behind the wing of the length aspect $\lambda = 2$, which begins to move from the state of rest with the velocity V = const = 1 at the angle of attack $\alpha = 20°$. We see that behind the wing there is formed an initial vortex which with elapsing time is worn out by the flow, and there are also formed vortex plaits close to the side portion of the shroud.

In Fig. 2 we have presented the dependence of the coefficients of the normal force c_n and the moment m_z of this force relative to the Oz axis, on the time t for a wing $\lambda = 2$, V = 1, $\alpha = 10°$. By dashed lines we have marked the values of these coefficients obtained according to the linear theory [7].

In Figs. 3 and 4 we have carried out a comparison of the experimental values of c_n and m_z with the corresponding theoretical values obtained on the basis of the method proposed in Secs. 2-4. The calculated quantities of the coefficients are shown by solid lines; dashed lines are used to show the values of the coefficients obtained on the basis of the linear theory [7]. Here we also have represented the experimental data [10, 11] (circles and triangles, respectively). We see that the theoretical curve $c_n(\alpha)$ for $\lambda = 2$ (Fig. 3) is close to the experimental data up to angles of attack $\alpha \sim 16$-$18°$, while the curve $m_z(\alpha)$ is so up to angles $\alpha \sim 11$-$12°$. The results of the calculation of $c_n(\lambda)$, $m_z(\lambda)$ for $\alpha = 10°$, without the side vortex shroud taken into account, well agree with the experimental data [10-11] and the data of [2, 9] in which a more general model with emergence of a side shroud in a fairly broad range of length aspects $(1 \leqslant \lambda \leqslant 4)$. Analogous calculations of $c_n(\lambda)$, $m_z(\lambda)$ for $\alpha = 15°$ showed a greater difference of the results obtained from the data [2, 9-11] for wings with the length aspect $\lambda < 1.5$. This is connected, apparently, with the fact that with a decrease in the length aspect and an increase in the angle of attack the role of the side vortex shroud in creating the lifting force and moments of the wing becomes dominant; stable vortex plaits are formed above the wing [3] which in fact improve the load-carrying properties of the wing.

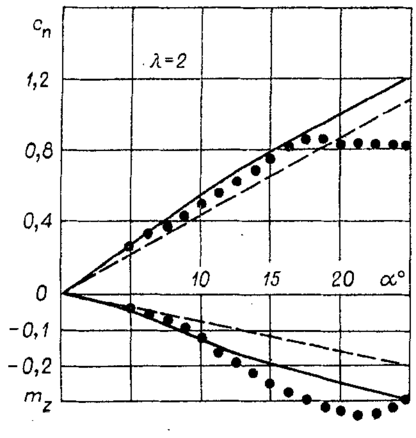

Fig. 3

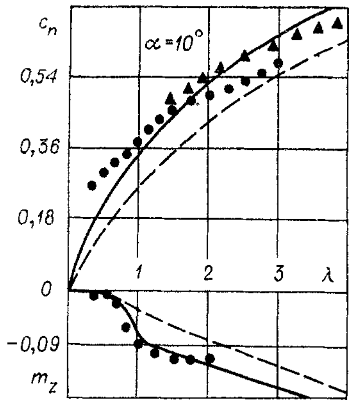

Fig. 4

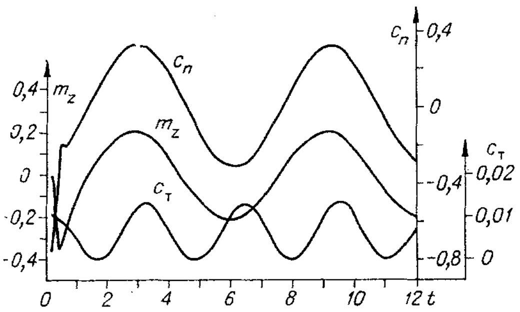

Fig. 5

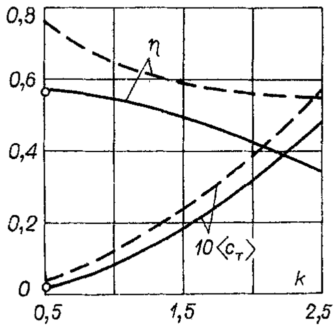

Fig. 6

About the results concerned with the stationary motion of the wing (V = const) we note the following: First, nonstationary values of the aerodynamic characteristics, as t increases, monotonically tend to certain values which are subsequently chosen as stationary values; second, such a calculation of these characteristics on the basis of the algorithm proposed above is fairly economical. For example, for a wing with the length aspect $\lambda = 2$ in the case of N = 25 and 30-40 steps in time, the calculation requires 6-8 min on a BÉSM-6 computer.

In the role of another example we present certain results of calculations with vibrating wings. When solving problems of flow around such wings, side by side with the coefficients c_n, m_z we determine the power used to maintain vibrations

$$N_0(t) = -\rho V_0^2 b^2 \lambda \iint_{S_0} \Delta p \dot{f} \, dx \, dz,$$

where $\dot{f}$ is the dimensionless velocity of the wing in the direction of the normal, and the drag coefficient $c_T = -c_q$. From values averaged over the period of vibrations $T = 2\pi/k$ ($k = \omega b/V_0$) $\langle N_0 \rangle$, $\langle c_T \rangle$, we determine the efficiency

$$\eta = \rho V_0^2 b^2 \lambda \langle c_T \rangle / (2 \langle N_0 \rangle).$$

In Figs. 5 and 6 we have represented the results of the calculation of a rectangular wing with the length aspect $\lambda = 2$ ($\alpha = 0$, $n_x = n_z = 5$), vibrating in the direction perpendicular to its plane according to the law $f(t) = 0.1 \sin kt$. In Fig. 5 we have presented the dependence of the coefficients c_n, m_z, c_T on time for the Strouhal number k = 1. The character of behavior of the curves in the case of $t < 1$ is explained by the influence of the transient processes at the beginning of motion from the state of rest. In Fig. 6 we have presented the dependence of the efficiency, and the drag coefficient $\langle c_T \rangle$ on the Strouhal number k. Here also by dashed lines we have recorded the values of these quantities calculated according to the linear theory for a wing of an infinite span [12]. Dots mark the quantities η, $\langle c_T \rangle$, obtained on the basis of the coefficients of aerodynamic derivatives of the inflow force presented in [13].

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INTERACTION OF THE WAKE OF A POORLY STREAMLINED BODY WITH A BARRIER

I. A. Belov

UDC 532.517.43

Continuing the work begun in [1], we investigate the problem of calculating the plane rotational flow of an ideal incompressible fluid near a plane barrier set up in a transverse position relative to the flow. Nonzero vorticity is induced in the outer flow by the formation of a wake after a poorly streamlined body placed in front of the barrier. We illustrate the solution of the stated problem in the example of the flow configuration created by uniform symmetric flow with velocity U past two parallel plates, one of which simulates the body, and the other the barrier.

For the analytical model of the flow past the plates we use the unsteady vortex model, which has been realized in practice by the method of discrete vortices [2] for the case of two plates of the same size (Ryabushinskii flow). Unlike the cited work, here we investigate flow past plates of different dimensions. The half-width of the second plate downstream is denoted by R, and that of the first by H, where $H < R$. The ratios between the plate dimensions H/R and L/R, were L is the distance between the plates, are adopted as the parameters to be varied.

An analysis of the vortex structures and fields of directions of the flow velocity vector in the wake of the plates for $H/R = 0.1$–1.0 and $L/R = 0.4$–2.2 shows that the flow cutoff

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